Name _____

AP Bio Math Review

Surviving $C_iV_i = C_fV_f$

C_i = original concentration of the solution, before it gets watered down or diluted

C_f = final concentration of the solution, after dilution

V_i = volume about to be diluted

V_f = final volume after dilution

By drawing the "X" through the equal sign and filling in the formula with letters of a size permitted by the borders of the "X", it reminds you that :

for all dilution problems $C_i > C_f$, and $V_i < V_f$

It makes sense because to dilute, we add water. This ***increases*** the volume but ***lowers*** concentration.

Examples by Type:

Easiest: Joe has a 2 g/L solution. He dilutes it and creates 3 L of a 1 g/L solution. How much of the original solution did he dilute?

$(2)(V_i) = (1)(3)$
$(2)(V_i) = 3$
$V_i = 1.5$ L

A little trickier: Joe has 20 L of a 2 g/L solution. He diluted it, and created 3 L of a 1 g/L solution. How did he make such a solution?

$(2)(V_i) = (1)(3)$
$(2)(V_i) = 3$
$V_i = 1.5$ L Joe only used 1.5 L of the 20 L

Trickier still: Joe has 20 L of a 2 g/L solution. To this solution he adds 30 L. What is the final concentration of the solution?

$(2)(20) = (C_f)(20 + 30)$
$40 = (50)(C_f)$
$C_f = 0.8$ g/L

pH Reminder: pH = -log (H^+)

Each pH unit represents a TENFOLD difference in H^+ and OH^- concentrations!

- Acidic solution: pH < 7
- Neutral solution: pH = 7
- Basic solution: pH > 7

- As pH decreases, acidity increases.
  - The LOWER the pH, the more ACIDIC a substance is
- As pH increases, acidity decreases.
  - The HIGHER the pH, the more BASIC a substance is

- As **[H^+]** gets smaller,
  - Scientific notation exponents get MORE NEGATIVE
  - pH goes UP
- As **[H^+]** gets larger,
  - Scientific notation exponents get LESS NEGATIVE
  - pH goes DOWN

Problem 1: What happens to the concentration of hydrogen ions [H+] vs hydroxide ions [OH-] when the pH of a solution is changed from 9 to 4?

More acidic

9 – 4 = 5

[H+] increased by 100,000 or 1 x 10^5

[OH-] decreased by 0.00001 or 1 x 10^{-5}

Problem 2: Rank the following from most acidic to least acidic.

pH= 10 pH = 6 pH = 2 $[H_3O^+] = 10^{-9}$ $[H^+] = 10^{-4}$ $[OH^-] = 10^{-6}$

pH = 9 pH = 4 pH = 14-6 = 8

Most acidic Least acidic

pH=2; $[H^+] = 10^{-4}$ or pH=4; pH=6; $[OH^-] = 10^{-6}$ or pH=8; $[H_3O^+] = 10^{-9}$ or pH=9; pH=10

Gibbs Free Energy

$\Delta G = \Delta H - T\,\Delta S$

What is Entropy? = a measurement of _____disorder_____

When **ΔS** is positive this means there is ____more disorder_____

When **ΔS** is negative this means there is ____less disorder_____

What is **ΔH**? = a measurement of ___enthalpy_____

When **ΔH** is positive this means the reaction is __endothermic (taking in energy)_____

When **ΔH** is negative this means the reaction is __exothermic (releasing energy)____

What is Gibbs Free energy? = a measurement of ____spontaneity____

When **ΔG** is positive this means the reaction will happen ___if energy is added__

When **ΔG** is negative this means the reaction will happen __spontaneously___

What happens to ΔG when ΔH goes up? WHY?
Goes up; more endothermic; needs more energy to occur; not spontaneous

What happens to ΔG when ΔH goes down? WHY?
Goes down; more exothermic; needs less energy to occur; more likely to be spontaneous

What happens to ΔG when T goes up? WHY?
Goes down; increase in temp increases kinetic energy; more collisions;
more likely to be spontaneous
What happens to ΔG when T goes down? WHY?
Goes up; decrease in temp decreases kinetic energy; less collisions;
less likely to be spontaneous
What happens to ΔG when ΔS goes up? WHY?
Goes down; more disorder; more likely to be spontaneous

What happens to ΔG when ΔS goes down? WHY?
Goes up; less disorder; less likely to be spontaneous

The hydrolysis of glucose-6-phosphate has ΔG = -4.0 kcal/mole (-16.5 kJ/mole) under standard conditions. Is this reaction

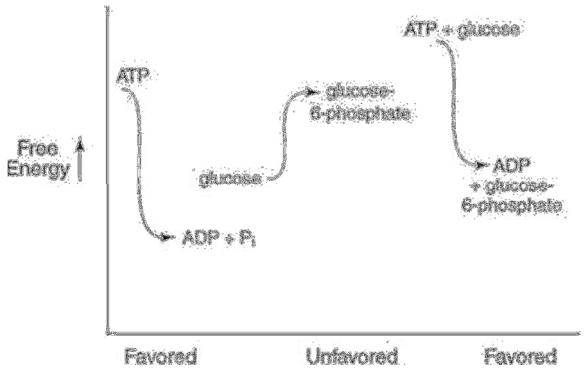
- Favored or unfavored favored
- Endergonic or exergonic exergonic
- Why? - ΔG

Although the opposite reaction is unfavored, the phosphorylation of glucose occurs readily in the cell, catalyzed by the enzyme hexokinase, because of energy coupling. An unfavorable reaction is driven by a favorable one.

The phosphorylation of glucose has ΔG = +4.0 kcal/mole (which is endergonic) but the hydrolysis of ATP is exergonic with ΔG = -7.3 kcal/mole. What is the standard free energy change of the coupled reactions?

4 + -7.3 = -3.3 kcal/mole (overall, - ΔG)

Coupling doesn't occur all by itself. In this example, if this experiment were set up so that the ATP would have to be hydrolyzed in one tube and the glucose phosphorylated in another, no coupling would be possible. Coupling can occur only when the partial reactions are part of a larger system. In this example, coupling occurs because both partial reactions are carried out by the enzyme hexokinase. In other cases, coupling can involve membrane transport, transfer of electrons by a common intermediate, or other processes. Another way of stating this principle is that coupled reactions must have some component in common.



The "orderliness" of your body is not favored by free energy. Explain (in terms of free energy and disorder) why you need to perform digestion?
Energy coupling: You digest food (natural tendency toward disorder) to harness that energy to build ATP (ordered), of which the breaking of ATP (natural tendency toward disorder) can be used to order your body
Why does decomposition of a dead animal happen in terms of energy? What would happen if we increase temperature? Why do we freeze food?
Decomposition is the breakdown of ordered cells into disorder. It should happen naturally as ΔG is negative. If you increase temp, ΔG becomes more negative, speeding up decomposition. We freeze food to LOWER temp, which increases ΔG, slowing down decomposition.
Explain why plant cells need light to build sugar (in terms of energy).
Building sugar (ordering molecules) would not happen spontaneously. Energy must be added (sunlight)

Surface Area to Volume Ratios

1. Determine the surface area-to-volume ratio for a cube that has a side length of 1cm compared to a cube that has a side length of 3cm. Which one is more efficient in terms of diffusion?

1cm: SA = 1 x 1 x 6 = 6cm^2 V= 1 x 1 x 1 = 1cm^3 SA:V = 6/1 = 6:1

3cm: SA = 3 x 3 x 6 = 54cm^2 V = 3 x 3 x 3 = 27cm^3 SA:V = 54/27 = 2:1

1 cm cube is more efficient because it has a higher SA:V ratio

2. Determine the surface area and volume the following 2 cells:

- Cell 1 (spherical) where the radius is 3 mm
- Cell 2 (rectangular) where the height is 0.5mm, length is 4mm, width is 2mm

	How to calculate Surface Area?	Surface Area	How to calculate Volume?	Volume
Cell 1	$4 \prod (3)^2$	113 mm^2	$(4/3) \prod (3)^3$	113 mm^3
Cell 2	2(2x0.5) + 2(4x0.5) + 2(2x4)	22 mm^2	L x W x H	4 mm^3

What is the surface area to volume ratio of each cell?

- Cell 1 113:113 = 1:1 (2:2)
- Cell 2 22:4 = 5.5: 1 (11:2)

Which cell is more efficient and WHY?

2 is more efficient Higher SA:V ratio

3. When agar is prepared using phenolphthalein and sodium hydroxide, the solidified agar has a pink color. Phenolphthalein is an indicator that is colorless below pH 10 but pink at any pH greater than 10. In a diffusion experiment, a phenolphthalein agar sphere (2 cm diameter) and phenolphthalein agar cube (each side 2 cm long) were placed in a dilute 0.1M hydrochloric acid solution for 10 minutes. Both shapes were gently agitated every minute to ensure the areas adjacent to each shape did not reach equilibrium. After 10 minutes both the sphere and the cube were cut in half and the colorless agar depth was measured to be 5mm.

- Calculate the diffusion rate of the hydrochloric acid for each shape
- If the 2 shapes were actual living cells, does one shape offer an advantage for the diffusion of nutrients into and wastes out of the cell? Explain and justify your answer mathematically

5mm / 10min = 0.5 mm/min for both the sphere and cube

Sphere: SA = $4\prod r^2 = 4\prod(1cm)^2 = 12.57cm^2$ V = $(4/3)\prod r^3 = (4/3)\prod (1\ cm)^3 = 4.19$

Cube: SA = 2 x 2 x 6 = 24cm^2 V = 2 x 2 x 2 = 8cm^3

12.57 / 4.19 = 3:1 24 / 8 = 3:1

Same efficiency because they have the same surface area to volume ratio and diffusion rate

Water and Solute Potentials

1. The value for Ψ in root tissue was found to be -3.3 bars. If you place the root tissue in a 0.1M solution of sucrose at 20°C in an open beaker, what is the Ψ of the solution, and in which direction would the net flow of water be?

$\Psi_S = -iCRT$
$\Psi_S = -(1)(0.1\ mol/L)(0.0831\ L*bars/mol*K)(293\ K) = -2.43\ bars$
$\Psi = \Psi_P + \Psi_S = 0\ bars + -2.43\ bars = -2.43\ bars$

The Ψ of the root tissue is -3.3 bars and the Ψ of the sucrose solution is -2.43 bars. Water will flow into the root tissue because free water always moves towards the lower overall water potential.

2. NaCl dissociates into 2 particles in water: Na^+ and Cl^-. If the solution above contained 0.1M NaCl instead of 0.1M sucrose, what is the Ψ of the solution, and in which direction would the net flow of water be?

$\Psi_S = -iCRT$
$\Psi_S = -(2)(0.1\ mol/L)(0.0831\ L*bars/mol*K)(293\ K) = -4.87\ bars$
$\Psi = \Psi_P + \Psi_S = 0\ bars + -4.87\ bars = -4.87\ bars$

The Ψ of the root tissue is -3.3 bars and the Ψ of the NaCl solution is -4.87 bars. Water will flow out of the root tissue and into the salt solution because free water always moves towards the lower overall water potential.

3. A plant cell with a Ψ_S of -7.5 bars keeps a constant volume when immersed in an open-beaker solution that has a Ψ_S 0f -4 bars. What is the cell's Ψ_P?

The plant cell keeps a constant volume because of the buildup of turgor pressure inside the cell. The Ψ_P at equilibrium would be the difference between the two solute potentials, which is 3.5 bars.

$-4 + 0 = x + -7.5$

4. At 20°C, a plant cell containing 0.6M glucose is in equilibrium with its surrounding solution containing 0.5M glucose in an open container. What is the cell's Ψ_P?

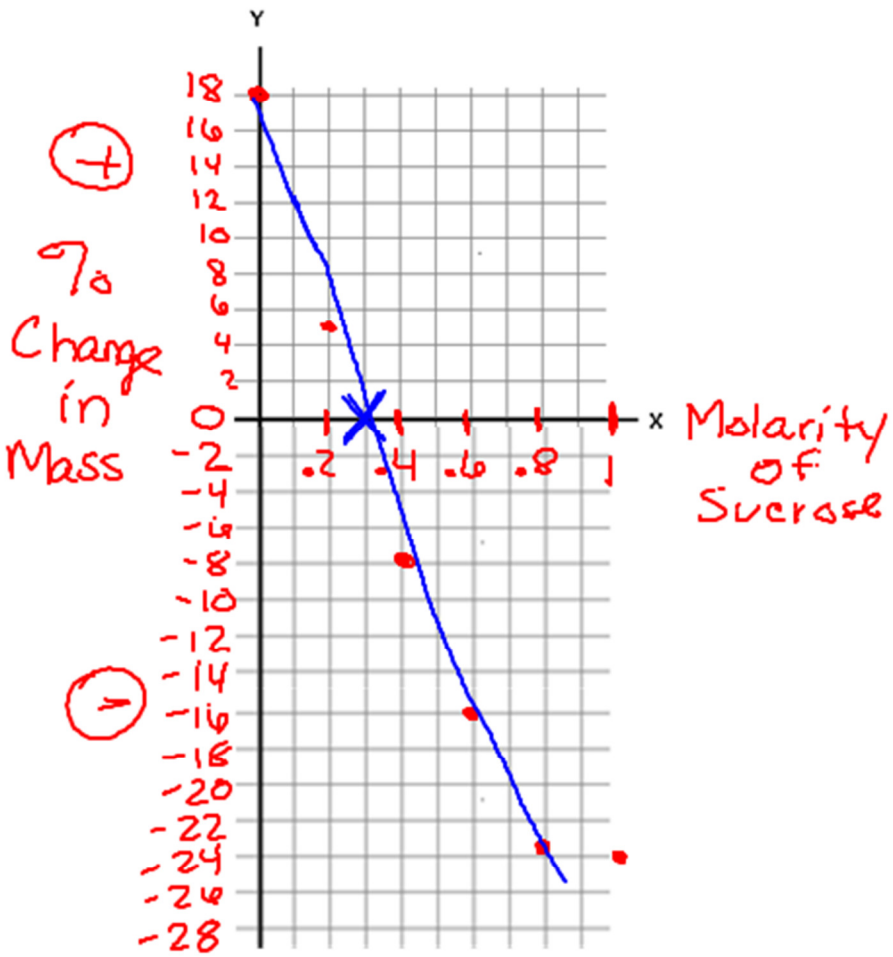
Surrounding solution: $\Psi = \Psi_P + \Psi_S =$ $0\ bars + -(1)(0.5\ mol/L)(0.0831\ L*bars/mol*K)(293\ K) = -12.2\ bars$

Cell at equilibrium: $-12.2\ bars = \Psi_P +$ $0\ bars + -(1)(0.6\ mol/L)(0.0831\ L*bars/mol*K)(293\ K) = -14.6\ bars$

$\Psi_P = -12.2\ bars - (-14.6\ bars) = 2.4\ bars$

5. Water potential in potato cells was determined in the following manner. The initial masses of six groups of potato cores were measured. The potato cores were placed in sucrose solutions of various molarities. The masses of the cores were measured again after 24 hours. Percent changes in mass were calculated. The results are shown below. Graph these data. Then determine the apparent molar concentration (osmolarity) of the potato core cells.

Molarity of Sucrose in Beaker	Percent Change in Mass
0.0 M	18.0
0.2	5.0
0.4	-8.0
0.6	-16.0
0.8	-23.5
1.0	-24.0



0.3 M

Population Growth

Example 1: There are 300 falcons living in a certain forest at the beginning of 2013. Suppose that every year there are 50 falcons born and 30 falcons that die.

a. What is the **population growth rate** (include units)? Interpret the value.

50 – 30 = 20 falcons/year
Each year, the falcon population will increase by 20 falcons

b. What is the **per capita growth rate** of the falcons over a year? Interpret the value.

20 = r (300)
r = 0.067
The falcon population will increase by 6.7% each year

c. Complete the table by determining the total population size each year from 2013 to 2018. Be sure to show your work.

Year	Population
2013	300
2014	318
2015	337.08
2016	357.3048
2017	378.7431
2018	401.468

300 x 0.06 = 18 300 + 18 = 318
318 x 0.06 = 19.08 318 + 19.08 = 337.08
337.08 x 0.06 = 20.2248 337.08 + 20.2248 = 357.3048

d. Find the **average rate of change** for the falcon population from 2013 to 2018.

Rate = Slope = change in y/ change in x

(401-300)/(2018-2013) = 101/5 = 20.2 falcons/year

Over the past 5 years, the falcon population has increased by 20.2 per year on average

Example 2: Kentwood, Michigan had a population of 49,000 in the year 2013. The infrastructure of the city allows for a carrying capacity of 60,000 people. r_{max} = .9 for Kentwood.

a. Is the current population above or below the carrying capacity? Will the population increase or decrease in the next year?

Below
Increase

b. What will be the **population growth rate** for 2013 (show your work and include units)?

(0.9)(49,000) [(60,000-49,000)/60,000] = 44,100 x 0.183 = 8085 people/year

c. What will be the **population size** at the start of 2014?

49,000 + 8085 = 57,085 people

d. Complete the following table by determining the population size and growth rate each year. Be sure to show your work.

Year	Population size	Population growth rate
2013	49,000	8085
2014	57,085	2496
2015	59,581	374.5
2016	59,955	40.5
2017	59,996	3.6

49,000 + 8085 = 57085

(0.9)(57,085) [(60,000-57,085)/60,000] = 2,496 people/year

e. What happened to the population growth rate as the population size approached carrying capacity?

Rate decreased

Hardy Weinberg

1. If 98 out of 200 individuals in a population express the recessive phenotype, what percent of the population would you predict would be heterozygotes?

(a) I have given you information on the frequency of the homozygous recessive (or q2). So start by determining q2 and then solving for q.

q^2 = (98/200) = 0.49 (or 49%)

q = square root of 0.49 = 0.7 (70%)

(b) Now that you have q, you can solve for p. Remember there are only two alleles in the population, so if you add the frequency of the two alleles, you have accounted for all possibilities and it must equal 1. So p + q = 1.

p = 1-q

p = 1 - 0.7 = 0.3 (30%)

(c) Now what is the formula for heterozygotes? Think back to the Hardy-Weinberg equation -- it is dealing with the genotypes of individuals in the population.

$p^2 + 2pq + q^2 = 1$

frequency of homozygous dominant + frequency of heterozygotes + frequency of homozygous recessive = 1

so.....2pq = frequency of heterozygotes

frequency of heterozygotes = 2 (0.3)(0.7) = 0.42 or 42%

2. Your original population of 200 was hit by a tidal wave and 100 organisms were wiped out, leaving 36 homozygous recessive out of the 100 survivors. If we assume that all individuals were equally likely to be wiped out, how did the tidal wave affect the predicted frequencies of the dominant and recessive alleles in the population?

Again, start with the frequency you know -- homozygous recessive. Follow the same step-by-step procedure as above.

q^2 = (36/100) = 0.36

q = square root of 0.36 = 0.6 (frequency of recessive allele)

p = 1 - 0.6 = 0.4 (frequency of dominant allele)

Given that the allele frequencies did change as the result of the tidal wave, we would say that microevolution has occurred.

Note: The drastic reduction in size of a population due to some chance event is called *a bottleneck event* - particularly when the original gene pool (allele frequencies) is no longer represented in the surviving population. Since there is now a small population, chances are likely that it will be subjected to *genetic drift* and continue to shift away from the original allele frequency (pre-tidal wave)

3. Let's say that brown fur coloring is dominant to gray fur coloring in mice. If you have 168 brown mice in a population of 200 mice, what is the frequency of each genotype?

Again start with what you know -- if 168 out of 200 are brown, that means 32 mice must be gray. There are only two phenotypes (three genotypes). So the first step is to determine q^2.

Why don't you start with p^2 since I given you the frequency of the dominant phenotype?

Think about it -- you have two DIFFERENT genotypes that can give you the dominant phenotype. Homozygous dominant and heterozygotes. So the frequency of the dominant phenotype would be:

$$\text{frequency of dominant phenotype} = p^2 + 2pq$$

You can't solve for p, if both p and q are unknown. So solve for q first ---

q^2 = (32/200) = 0.16 (frequency of homozygous recessive)

q = 0.4

p = 1 - (0.4) = 0.6

p^2 = (0.6)(0.6) = 0..36 (frequency of homozygous dominant)

2pq = 2 (0.6)(0.4) = 0.48 (frequency of heterozygotes)

Double check!

$p^2 + 2pq + q^2 = 1$

0.16 + 0.48 + 0.36 = 1.0

Chi Square

1. At a particular high school, students can choose to enter one of three doors. Custodians noticed that door #3 was always getting broken and suggested that more students use that door because it has a hands-free opener. Science minded students counted the number of students entering each door to see if the custodians were right.

- Door #1 had 60 students enter
- Door #2 had 66 students enter
- Door #3 had 80 students enter.

Were the custodians right? Use Chi Square analysis to support your answer.

Null = no sig difference between number of individuals using each door (any diff due to chance)

60 + 66 + 80 = 206 total 206 / 3 = 69 expected

$(60-69)^2 / 69 = 81/69 = 1.17$
$(66-69)^2 / 69 = 9/69 = 0.13$
$(80-69)^2 / 69 = 121/69 = 1.75$

1.17 + 0.13 + 1.75 = 3.05

	1	2	3
Exp	69	69	69
Obs	60	66	80

DF = 3-1 = 2
Critical value = 5.99

Chi square value is less than critical value so we accept the null (chance)
Custodians are wrong

2. In peas, yellow seeds (A) are dominant over green seeds (a). In a cross between two plants that are both heterozygous for seed color, the following was observed:

- Yellow = 4400
- Green = 1624

Does the data fit the predicted ratio? Use Chi Square analysis to support your answer.

A = yellow
a = green

♀ Aa ♂ Aa

♀: A, a ♂: A, a

♀ \ ♂	A	a
A	AA	Aa
a	Aa	aa

expected phenotypic ratio
3 : 1
yellow : green

Phenotype	Obseved (O)	Expected (E)	O-E	(O-E)²	(O-E)²/E
Yellow	4400	6024(.75) = 4518	-118	13924	3.08 = 13924/4518
Green	1624	6024(.25) = 1506	118	13924	9.24 = 13924/1506
Totals	6024	6024			12.32

n = 2 - 1 = 1
P = 3.84

No, data does not fit predicted phenotype as 12.32 > 3.84

POOR FIT

3. Color blindness is a sex-linked trait in Bombats. A female who is a carrier of the color blind allele mates with a male who is color blind. The phenotypes of their offspring are:

- Normal female = 132
- Color blind female = 124
- Normal male = 126
- Color blind male = 136

Does the data fit your predicted phenotypic ratio?

Normal female = 132
Color blind female = 124
Normal male = 126
Color blind male = 136

X^B = normal
X^b = color blind
Y = male

♀ X^BX^b ♂ X^bY

♀: X^B, X^b ♂: X^b, Y

♀ \ ♂	X^b	Y
X^B	X^BX^b	X^BY
X^b	X^bX^b	X^bY

Expected Phenotypic Ratio

Normal ♀ = 1 = 0.25
Colorblind ♀ = 1 = 0.25
Normal ♂ = 1 = 0.25
Colorblind ♂ = 1 = 0.25

Phenotype	Obseved (O)	Expected (E)	O-E	(O-E)²	(O-E)²/E
Normal ♀	132	518(.25) = 129	3	9	16/129 = 0.07
colorblind ♀	124	129	-5	25	25/129 = 0.19
Normal ♂	126	129	-3	9	9/129 = 0.07
colorblind ♀	136	129	7	49	49/129 = 0.38
Totals	518	~ 518			0.71

n = 4 - 1 = 3
P = 7.82

Yes, the data fit the predicted phenotype 0.71 < 7.82

GOOD FIT

4. In one particular species of plant, purple flowers (P) are dominant over red flowers (p) and long pollen grains (L) are dominant over round pollen grains (l). After crossing true breeding plants that have purple flowers and long pollen grains with true breeding plants that have red flowers and round pollen grains, the resulting dihybrids (PpLl) were then crossed. Of the resulting 381offspring, 284 had purple flowers and long pollen grains, 21 had purple flowers and round pollen grains, 21 had red flowers and long pollen grains, and 55 had red flowers and round pollen grains. Determine whether or not the genes for flower color and pollen grain shape are linked or unlinked. Use Chi Square analysis to support your answer.

Null Hypothesis:

- If the two genes are unlinked, we would expect a phenotypic ratio of 9:3:3:1.

Calculate your Chi-Squared Value:

- To calculate expected results:
  - What is the probability that the offspring would have each phenotype?
    9/16 purple/long : 3/16 purple/round:
    3/16 red/long: 1/16 red/round
  - Multiply this fraction by the total number of offspring produced
    381 x (9/16) = 215
    381 x (3/16) = 71
    381 x (1/16) = 24

- Calculate your chi-square value
  - Show your work:
    22.14 + 35.21 + 35.21 + 40.04
  - Chi-square value = 132.6

	P/Long	P/Round	R/Long	R/Round
Exp	215	71	71	24
Obs	284	21	21	55

- Determine the critical chi-square value (use a p-value of 0.05)
  - Degrees of freedom = 4 – 1 = 3
  - Critical chi-square value = 7.82

Draw your Conclusion

- Our calculated chi-square value is more than our critical chi-square value
- Therefore, we reject our null hypothesis which means
  - The difference between our observed numbers and our expected numbers IS statistically significant
  - Our predicted inheritance pattern is a POOR FIT for the data
    - The genes do NOT assort independently and may therefore be linked